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MARFE onset conditions and stability

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Abstract

The MARFE stability is considered on closed magnetic field lines in a configuration having an X-point. The model incorporates both perpendicular and parallel transport and takes into account the flux expansion in the vicinity of the X-point. The stability analysis can be reduced to a standard eigenvalue problem for the temperature perturbation. The ballooning representation of the perturbation is shown to be essentially a method to separate radial and poloidal variables and to reduce the two-dimensional heat conduction equation with periodic metric coefficients to a one-dimensional equation in a ballooning space. The effect of the toroidal magnetic topology on the marfe stability is investigated. It is shown that the flux expansion near the X-point has a destabilizing effect. The competing stabilizing effect is associated with the 'steepening' of the ballooning perturbation both in radial direction (between the magnetic surfaces) and perpendicular to the magnetic field lines on the magnetic surfaces. The main result of the work argues that in the stability analyses both parallel and perpendicular heat fluxes must be taken into account and can not be omitted without change in the spectrum of the anisotropic heat conduction equation.

Keywords: Divertor modeling; Divertor plasma; ITER; MARFE

1. Introduction

The MARFE phenomenon is usually quoted as an example of the self-organisation processes in a tokamak edge plasma [1]. Self-enhanced impurity radiation cooling still remains the main physical mechanism for the thermal-radiation instability of the impure edge plasma. This instability develops when a local decrease of temperature re-enforces the impurity radiation, causing a further cooling of the plasma, and when thermal conduction is unable to compensate this energy loss. The experimental observations from ASDEX Up and other tokamaks indicate that the marfe tends to locate itself near to the X-point, where it can be almost in quasi-steady state condition [2].

The linear stability analyses of the plasma edge parameters which provides the onset of the X-point MARFE should be carried out in a 2D toroidal magnetic geometry with an X-point. The main problem of a such 2D analysis in the toroidal geometry is that the variables (usually they are the flux co-ordinates) are not separable, thus one is unable to apply the usual representation involving the ordinary Fourier expansion without having a mixture of the eigenmodes. The problem arises due to the poloidal variation of the metric coefficients and the equilibrium quantities.

A number of investigations have been performed in this direction, reducing the 2D problem to a 1D problem by simply excluding the radial or poloidal heat flux in the heat equation or by considering the cylindrical approximation, thus ignoring the toroidal and X-point effects [3]. In some consideration the perpendicular heat fluxes were excluded because of the high classical electron heat conduction along the field lines, enforcing nearly constant temperature on magnetic flux surfaces. However, because of the strong temperature dependence of the classical parallel conduction and the electron heat flux limit at low densities, noticeable gradients along field lines are to be expected at typical tokamak edge parameters if there are sufficiently strong, localized

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energy sinks. These can be caused by, for example, impurity radiation. It is obvious that this simplification does not work close to the X-point, where the radial fluxes are expected to be strong. For the same reason the toroidal effects must be fully employed in the stability analyses of the MARFE-type perturbation in a realistic tokamak configuration. The numerical attempts to solve the 2D heat equation in realistic geometry brings with it a complicated mixture of the modes that makes the analysis rather difficult [4].

In this paper we consider a 2D linear stability analysis of the MARFE-type perturbation inside the last magnetic surface in a toroidal geometry with a separatrix. Based on the special type of perturbation which allows one to resolve the separability problem of the heat equation in toroidal geometry, we will prove that both the radial and parallel heat fluxes should be taken into account and cannot be omitted without change of the spectral properties of the anisotropic heat conduction equation. The separation of variables can be strictly performed in case of the 2D toroidal geometry including the X-point by employing a so-called 'ballooning type' of perturbation. The ballooning representation has been first invented to overcome the same difficulty in the ballooning equation of the MHD perturbation in the toroidal geometry [5]. Below we will show that this type of perturbation, being applied to the heat equation can resolve the problem of the separability of the variables and to provide the analyses of the onset conditions without any 'simplified' suggestions, corrupting the operator of the 2D differential equation.

2. Equation and topology

We begin the linear stability analyses by considering the heat equation in orthogonal flux co-ordinate system and assuming the constant pressure along the magnetic field lines:

$$\dot{\varepsilon} + \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^i}(\sqrt{g}q^i) = S_\varepsilon \quad \text{and} \quad (\vec{B}\,\nabla_p) = 0, \tag{1}$$

where

$$q^i \equiv -\chi_\perp g^{ik}\frac{\partial T}{\partial x^k} - (\chi_{//} - \chi_\perp)b^i\left(b^k\frac{\partial T}{\partial x^k}\right), \quad |\vec{b}^2| = 1. \tag{2}$$

Here b^i and q^i are the covariant components of a unit vector in the direction of the magnetic field, $\vec{b} = \vec{B}/|\vec{B}|$, and a heat flux component, respectively. In Eq. (2) we took into account the different heat conduction coefficients along B, $\chi_\parallel$ and across the magnetic surfaces, $\chi_\perp$. The source term in Eq. (1) arises from impurity radiation, $S_\varepsilon = -n^2 c_z L(T)$, where n is a plasma density, $L(T)$ is a cooling rate function and c_z is an impurity concentration. The rest of the definitions are obvious. We aim to consider the linear stability of the Eq. (1) in toroidal geometry close to the separatrix area. For this purpose we choose the orthogonal flux co-ordinate system allowing for a plasma shape with X-point. For simplicity we choose the topology created by a pair of parallel wires carrying equal currents [6]. The model possesses a separatrix, with an X-point midway between the wires and allows one to investigate thermal stability at various distances from the separatrix and to examine the effect on marginal stability when changing the location (in poloidal angle) of the X-point. The metric coefficients can be expressed analytically. The line element in this case reads as: $ds^2 = h^2(d\rho^2 + d\theta^2) + R^2\,d\varphi^2$, where ρ is marking the magnetic surfaces, and θ and φ are the poloidal and toroidal angular variables. Here $g_{\psi\psi} = g_{\theta\theta} \equiv h^2$, $g_{\varphi\varphi} = R^2$, $\sqrt{g} = h^2 R(\theta, \varphi)$ and $h^2 = e^{2\rho}/4y_0^2 b^2$. The major radius for the current point position at the surface is $R(\theta, \rho) = R_0/y_0 \pm \sqrt{(a+b)/2}$, R_0 is the distance from the azimuthal axes to the current position at the mid-plane, y_0 is the distance of the current wire from the X-point position (see Fig. 1). Here $a \equiv e^\rho \cos\theta - 1$ and $b^2 \equiv 1 - 2e^\rho \cos\theta + e^{2\rho}$. We shall consider the surfaces lying inside the separatrix and they are labelled by a parameter ρ, such that when $\rho \to -\infty$ the surfaces become circular. As $\rho \to 0$, the shape of the surfaces approaches that of a separatrix and $h \propto 1/\rho$. For numerical convenience we will use below another parameter for labelling the surfaces, ξ, which is linearly shifted relative to ρ, $\xi \equiv k\rho + \text{const}$. Here $k \approx 7.9$. ξ ranges from $-\infty$ at the core area to some positive value $\xi = \xi_{sep}(\xi_{sep} \approx 0.68)$ at the separatrix. The ψ_{95} distance corresponds to about $\xi_{95} \approx 0.6$. The poloidal magnetic field caused by straight currents and the toroidal magnetic field can be

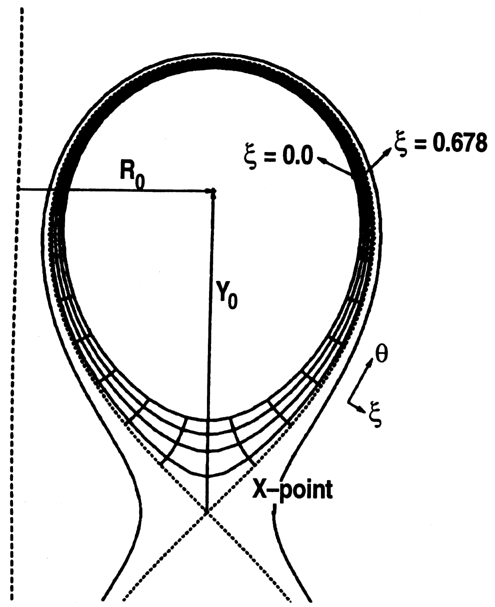

Fig. 1. Co-ordinate system allowing for the X-point.

chosen as $b_\theta = \gamma/\sqrt{h^2+\gamma^2}$, $b_\varphi = \sqrt{1-b_\theta^2}$, $b_\rho = 0$, where $b_\theta \equiv B_\theta/B$, $b_\varphi \equiv B_\varphi/B$ are the physical components and $\gamma \approx$ const. is taken to mach the ITER magnetic field.

Eq. (1) in the orthogonal co-ordinates read:

$$\frac{\partial}{\partial\theta}\{h^2Rq^\theta\} + k\frac{\partial}{\partial\xi}\{h^2Rq^\xi\} + \frac{\partial}{\partial\varphi}\{h^2Rq^\varphi\} = -n^2c_zL(T)h^2R \tag{3}$$

and $p \approx$ const. on the magnetic surfaces. Here

$$q^\theta \equiv -\frac{\chi_\theta}{h_\theta^2}\frac{\partial T}{\partial\theta} - (\chi_{//}-\chi_\perp)\frac{b_\theta}{h_\theta}\frac{b_\varphi}{R}\frac{\partial T}{\partial\varphi}; \tag{4}$$

$$q^\xi \equiv -\frac{\chi_\perp}{h_\rho^2}\frac{\partial T}{\partial\xi}; \tag{5}$$

$$q^\varphi \equiv -\frac{\chi_\varphi}{R^2}\frac{\partial T}{\partial\varphi} - (\chi_{//}-\chi_\perp)\frac{b_\theta}{h_\theta}\frac{b_\varphi}{R}\frac{\partial T}{\partial\vartheta}; \tag{6}$$

here $\chi_\vartheta \equiv \chi_\perp + (\chi_{//}-\chi_\perp)b_\theta^2$ and $\chi_\varphi \equiv \chi_\perp + (\chi_{//}-\chi_\perp)b_\varphi^2$.

In equilibrium, due to the toroidal symmetry of the problem we can omit the third term in Eq. (4). The equation reveals several equilibrium solutions which may be classified as those which have a constant temperature along the field line (MARFE-free, radial equilibrium) and to a MARFE equilibrium when the temperature varies along the field line, i.e. exhibit a MARFE-like character. The first case can also be considered as a poloidally symmetric radiating region on closed flux surfaces (detached MARFE) and its linear stability against the most unstable poloidal mode has been treated in [3] as an eigenvalue problem, ignoring the dependence of the metric coefficients on θ. In the geometry adopted here a poloidally symmetric equilibrium reads as a balance of the radial heat fluxes incoming to and outgoing from the poloidal layer:

$$\left\{Rq_\xi\frac{h}{k}\right\}_\xi^2 = \left\{Rq_\xi\frac{h}{k}\right\}_\infty^2 - \int_\infty^T 2n^2c_zL(T)\left(\frac{hR}{k}\right)^2\chi_\perp\ \mathrm{d}T,$$

where $q_\xi\frac{h}{k}\mathrm{d}\xi = -\chi_\perp\ \mathrm{d}T$ (7)

this implies that the equilibrium can not be only a function of the minor radial co-ordinate (as it taken in [3]), but it varies with poloidal angle. Another equilibrium corresponds to the temperature variation along the field line and can be caused by a strong radiative cooling due to impurity accumulation in the vicinity of the X-point. The stability analyses in the last case defines which temperature gradients are consistent with the MARFE location at the X-point.

The stability analyses of Eq. (1), with the periodic boundary conditions in poloidal direction on the closed magnetic field lines, must be treated as an eigenvalue problem for the parabolic partial differential equation. In the above equation the thermal coefficients are taken as a function of temperature and density: $\chi_{//} = \chi_{//0}T^{5/2}$, $\chi_\perp = \chi_{\perp 0}nT$, where $\chi_{//0}$ and $\chi_{\perp 0}$ are some constant values. We require the solution to be 2π-periodic in θ, φ space and vanish at infinity with respect to ξ. Obviously the operator in Eq. (3) is not separable as it stands. Following [5] we consider Eq. (3) in "ballooning space" – which is the extended infinite θ domain and try a temperature perturbation of the form

$$T(\theta,\xi,\varphi) = W(y)\mathrm{e}^{\mathrm{i}m(\varphi - \int_0^y q(\xi,t))\ \mathrm{d}t}, \tag{8}$$

where $y \equiv \theta - \theta_0$, is a toroidal mode number and $q(\xi, y) \equiv (h_\theta/R)(b_\varphi/b_\theta)$. The trial function (8) corresponds to a perturbation with a long parallel wavelength and short perpendicular wavelength with a large harmonic number $mq \gg 1$. θ_0 is a free parameter in the ballooning presentation. The perturbation (8) enables us to separate variables and brings us to the following 1D Schrödinger-type equation for $\Phi(y)$:

$$\Phi_y'' = U(y,\xi,m,\gamma)\Phi, \tag{9}$$

where $\Phi \equiv W/b_\theta\sqrt{R}$ and for the potential well we have:

$$U(y,\xi,m,\gamma) \equiv U_m(y,\xi,m) + U_z(y,\xi,\gamma) + U_\theta(y,\xi,\nabla T_0), \tag{10a}$$

$$U_m(y,\xi,m) \equiv m^2\frac{\chi_\perp}{\chi_{//}b_\theta^2}\left(\frac{h_\theta^2}{R^2b_\theta^2} + I_\xi^2\right), \tag{10b}$$

$$U_z(y,\xi,\gamma) = \gamma\frac{h^2}{\chi_{//}b_\theta^2}\frac{\partial}{\partial T}\left(\frac{L(T)}{T^2}\right), \tag{10c}$$

$$U_\theta(y,\xi,\nabla T_0) = \frac{1}{4}\left(\left(\frac{\partial\ln(R\chi_\theta)}{\partial y} + \frac{\partial\ln\chi_\theta}{\partial T}\left(\frac{\partial T}{\partial y}\right)_0\right)\right)^2 + \frac{1}{2}\frac{\partial}{\partial y}\left(\left(\frac{\partial\ln(R\chi_\theta)}{\partial y} + \frac{\partial\ln\chi_\theta}{\partial T}\left(\frac{\partial T}{\partial y}\right)_0\right)\right) - \frac{1}{R\chi_\theta}\left[\frac{\partial}{\partial y}\left(R\left(\frac{\partial\chi_\theta}{\partial T}\right)\frac{\partial T}{\partial y}\right)\right]_0. \tag{10d}$$

Here $\gamma \equiv p^2c_z$ is an eigenvalue and we denote the equilibrium terms in Eq. (10d) by prescribing index 0.

Eq. (9) is an ordinary 1D differential equation, which can be easily analysed and solved numerically, assuming that a new independent variable y varies in the infinite domain. The boundary conditions are now: $\Phi(\pm\infty) = 0$, $y \in (-\infty, \infty)$. The basic idea of the chosen transformation is that the spectrum of this eigenvalue problem in the infinite θ range is the same as in the original Eq. (3) in the periodic poloidal domain [5]. Following the property of the ballooning modes only terms of the order of m^2 remained in Eq. (9).

Below the eigenvalue problem (9)–(10d) has been solved numerically. The domain of integration of 5π in poloidal angle was found to be adequate. As a reference we took the ITER-like parameters ($R = 8$ m, $B = 5.6$ T etc.). Argon has been taken as an impurity sample and the cooling rate $L(T)$ from [7] has been employed, assuming a non-coronal radiative equilibrium. The second equation in (1) for pressure balance along the magnetic field assumes that the pressure perturbation $p' \approx 0$, and that impurities are following the perturbations of the plasma ions.

3. Analysis

The first term U_m in the expression for the potential well (10a)–(10d) is always positive. It represents the toroidal part of the perturbation and is attributed to a stabilizing role of the perpendicular (to the magnetic field lines) heat fluxes both along the magnetic surfaces ($\propto h_\theta^2/R^2 b_\theta^2$) and across the surfaces ($I_\xi \equiv k \int_0^y \partial q/\partial\xi \, dy$). The second term U_z is a destabilizing term and is attributed to the thermal instability. This term creates a negative potential due to a negative slope of the cooling rate function in the corresponding temperature domain. The rest of the terms in U_θ are associated both with the poloidal variation of the magnetic topology (volume element and the parallel heat flux) and with the equilibrium temperature gradient along the field lines. The stabilizing effect of the parallel heat flow reveals itself in denominators of all terms, which contain the value $\chi_\theta \approx \chi_{//} b_\theta^2$, so that the contribution of all terms (stabilizing or destabilizing) in the potential well is normalized to that of the parallel heat conductivity. In the vicinity of the X-point the flux expansion and the vanishing of the poloidal projection of the parallel heat flux should diminish the stabilizing effect as ($b_\theta^2/h^2 \to \rho^{-4}(\rho \to 0)$ and $U_m \to 0$ at the separatrix. Analysis shows that the poloidal variations in U_θ make the stability conditions more complicated.

First we consider the toroidally symmetric temperature perturbations ($m = 0$). Fig. 2 shows the stability diagram for such perturbations at the magnetic surface position $\xi = 0.6$, which corresponds roughly to ψ_{95}. The stability diagram has two regions, the region above the marginal value of $\gamma = p^2 c_z$ which is unstable to marfes and the region below this value where the temperature perturbation become stable. The marginal γ increases as expected for higher temperatures. Figs. 3 and 4 show the corresponding eigenfunction and potential behaviour vs. poloidal angle for $T = 100$ eV. Numbers on figures indicate: 1 for U_m terms, 2 for U_z, 3 for the terms in U_θ proportional to the equilibrium temperature gradient, 4 and 5 represent the first terms in U_θ. The dashed line shows the resulting potential. It is interesting to note, that at $\theta = 0$ the potential has a maximum (the eigenfunction passes through the minimum (see Fig. 3)) and the negative part of the well is shifted symmetrically away from the X-point. This indicates that the perturbations are more stable at the X-point, than expected. This result is attributed to the poloidal variation of the coefficient in U_θ, namely: $1/2(\partial/\partial\theta)(\partial \ln (Rb_\theta^2)/\partial\theta)$ which overcomes the negative contribution from U_z at the X-point and creates two negative wells in neighbouring positions to the X-point. The closer to the separatrix the well is located the deeper it becomes, however, in reality its shape and deepness do not change much, because they are limited by a similar term as the one in U_θ: $1/4(\partial \ln (Rb_\theta^2)/\partial\theta)$ which gives a positive contribution. This is the reason, why the perturbations become almost insensible to the radial position from the separatrix (see Fig. 5), except in the very vicinity to X-point.

The stability of the toroidal perturbations ($m \neq 0$) is shown in the Figs. 6 and 7. The critical impurity content, required for a MARFE onset can be found for each

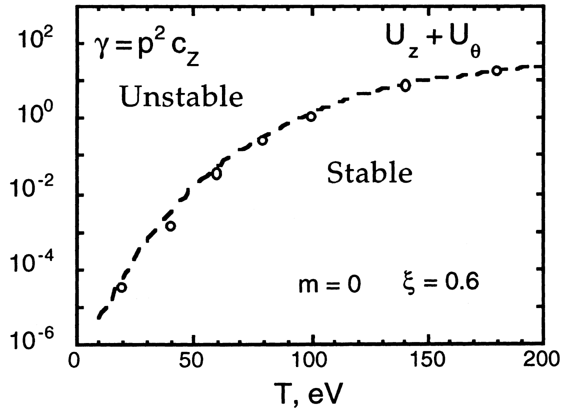

Fig. 2. Stability diagram for toroidally symmetric temperature perturbation on the magnetic surface $\xi = 0.6$ (95%); toroidally mode number $m = 0$.

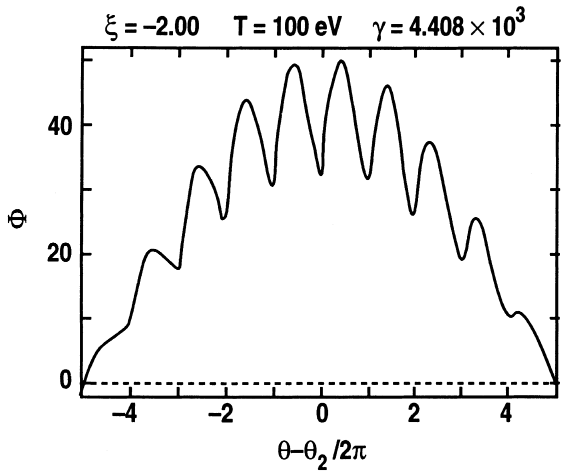

Fig. 3. The eigenfunction for $m = 0$; $T = 100$ eV.

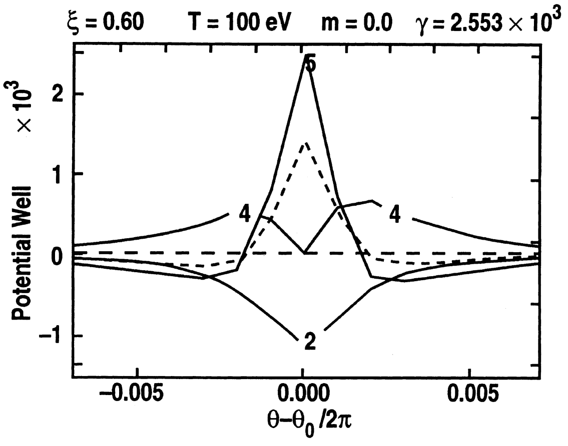

Fig. 4. Potential U vs. poloidal angle; toroidally symmetric perturbation, $m = 0$; $T = 100$ eV. Dashed line is a sum of all terms; $U_m = 0$.

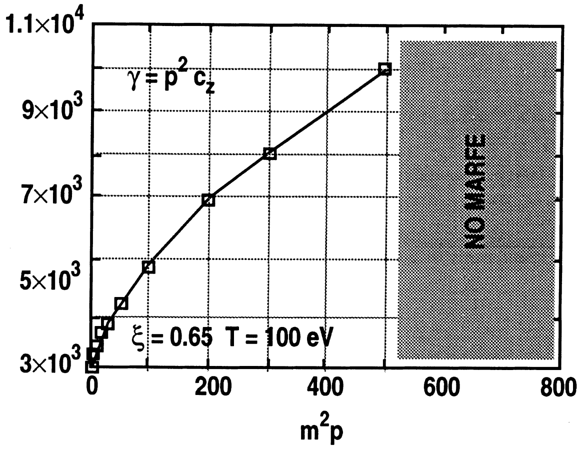

Fig. 6. Stability diagram for toroidal temperature perturbation on the magnetic surface, $m > 1$; $\xi = 0.65$. Above some critical m^2p the perturbation does not exist any more.

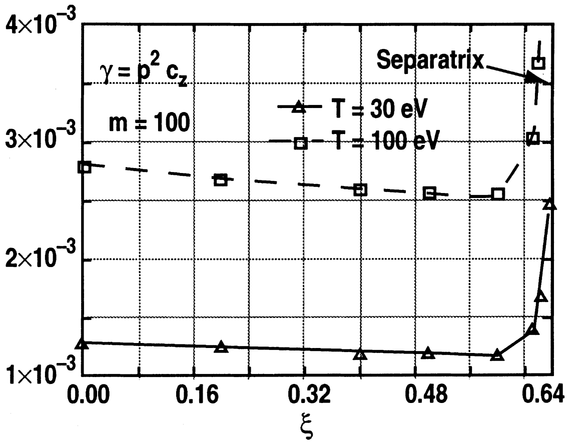

Fig. 5. Marginal value p^2c_z on the different magnetic surfaces, $m = 100$; for $T = 30$ eV and $T = 100$ eV. The most unstable position is slightly above the X-point location, which corresponds to the minimum $\gamma = p^2c_z$.

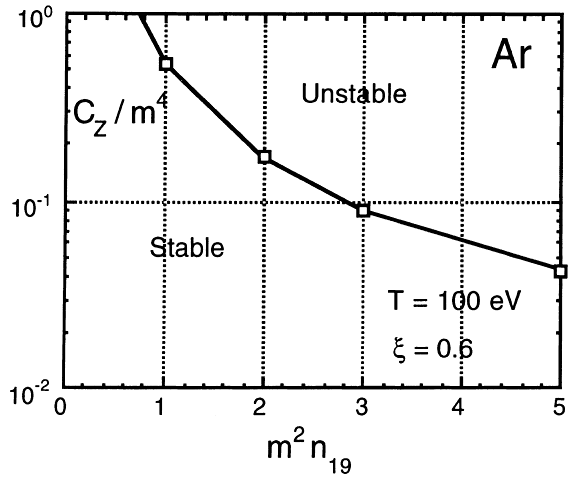

Fig. 7. Ar concentration vs. plasma density at given toroidal mode number m, $T = 100$ eV, $\xi = 0.6$.

toroidal mode number m (at given plasma density or pressure). The perturbations of this type become more stable due to the stabilizing role of the perpendicular heat fluxes. They are strongly stabilized especially near the X-point, where the perturbations on each magnetic field line approaches each other, resulting in strong gradients both across and along the surfaces. This increases the fluxes and brings about the stabilization. Far from the X-point position, the shape of the potential well becomes more shallow (see Fig. 8) due to the positive contribution of U_m.

We also investigated how sensitive the stability is against the poloidal variation of the temperature. We chose the equilibrium temperature profile along the field line as $T(\theta) = T_1 - T_2\cos\theta$, where T_1 is some average temperature. By varying T_2 we find that the potential well (being mostly affected by U_z) becomes negative and centred at the X-point, whereas the contribution of the rest terms in U_0 is negligible. This effect of destabilization (the increasing of U_z) is mainly due to the lowering of $\chi_\theta(T, \theta)$ at low temperatures.

4. Conclusions

The main results are the following. The 2D linear stability problem of a MARFE-like temperature perturbation on closed magnetic surfaces has been reduced

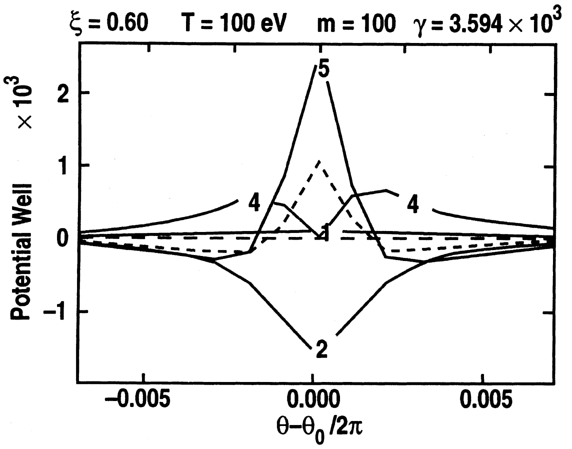

Fig. 8. Potential U vs. poloidal angle; toroidally nonsymmetric perturbation, $m = 100$; $T = 100$ eV.

to a 1D eigenvalue problem using the ballooning representation for the perturbation. This type of a perturbation has a long parallel wavelength and short perpendicular wavelength typical for ballooning modes. The toroidally inhomogeneous temperature perturbations with non-zero mode toroidal numbers, m, having a weak variation along the magnetic field lines have been analysed and compared with toroidally symmetric temperature perturbations ($m = 0$). The trial functions for the temperature perturbations for both ($m = 0$) and ($m > 1$) cases are localized on the closed magnetic surfaces close to the X-point. The toroidal mode numbers m of marginally stable perturbations were found as a function of impurity concentration (at given plasma density or pressure). The geometry effects (variation of the metric coefficients with poloidal angle) have a strong influence on stability, ensuring localization of the marfe-type perturbation slightly above the X-point.

Finally it should be noted that here only one type of the possible MARFE-like perturbation was considered, namely the ballooning type, allowing one to reduce the problem to a 1D equation. These perturbations are localized on the magnetic surfaces and have a similar structure on each neighbouring surfaces. However, there could be a variety of other type of perturbations, which can also be related to the thermal stability problem and can involve different sources of free energy. We choose the ballooning modes just for illustrative purposes in order to show that the property of the heat operator can be easily violated by arbitrary ignoring its 2D structure. It is especially a concerns for the X-point area where the spectral analyses becomes complicated

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